Automatically Synthesised Selection Algorithm

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Theorem 1. $V_4(7) \leq 10$

Proof. Algorithm 1 presents a method for computing the 4 -th largest of 7 elements. In the worst-case, the number of comparisons is equal to 10. □

Algorithm 1. *Let KEYS be a totally ordered set, |KEYS| =7.*

1. *Partition the set KEYS into disjoint subsets |K1|=4, |K2|=2, |K3|=1. Determine the maxima of K1, K2, K3 by setting-up balanced tournaments. The resulting poset is isomorphic to the poset as shown in Figure 1. For setting-up the balanced tournaments 4 comparisons are needed.*

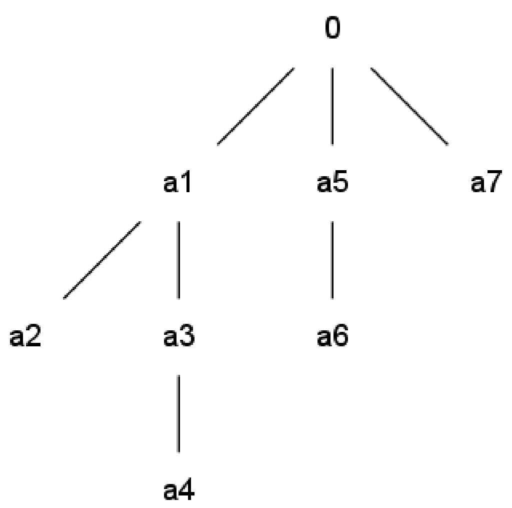

Figure 1: Balanced Tournaments of Set Partition

2. (a) *Perform the comparisons in accordance with the decision graph in Figure 2*[1]

[1] The nodes in the decision graph represent:

i. Comparisons X<Y, shown as circles.Non-filled arrows represent the case X>Y and filled arrows refer to the case X<Y.

ii. Subgraph place holders presented by diamonds, which correspond to solutions which can be obtained as instances from theorems and algorithms known from published literature.

iii. Place holders for calls to isomorphism functions, which are denoted by double circles. There are two type of function calls. Firstly, solutions for the isomorphic subcases are part of the presented proof graph and.are being refer<enced. Secondly, in some situations more detailed explanations for the isomorphic subcases have been generated. The referenced cases are indicated by circles.

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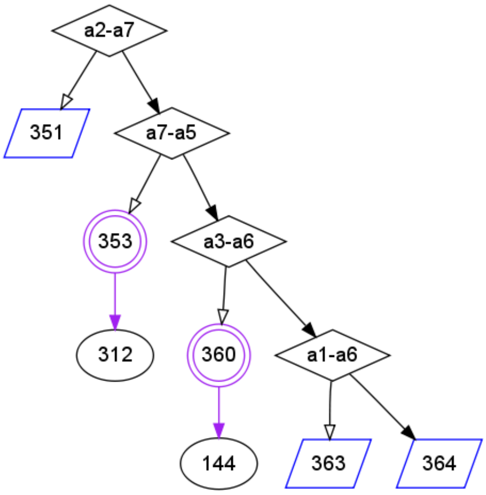

Figure 2: decision graph

(b) *Further steps correspond up to isomorphism with solutions which are retrieved from the case base.*

Node 144 refers to Case 1.

Node 312 refers to Case 2.

(c) *Solutions for the subproblems associated with the nodes* 351,363,364 *may be obtained as instances of the Algorithms* Aigner (1982) Kislitsyn (1964) .

Case 1. *Let P be the partially ordered set as visualised in Figure 3. The 3-rd largest element of P can be computed by at most 3 comparisons.*

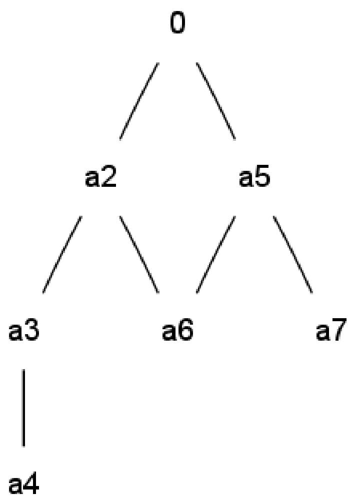

Figure 3: Poset P

Proof. Compare a3 and a7.

if *a3* > *a7* **then**

Selecting the 2nd largest element takes at most f2(1,1)=2 further comparisons Kislitsyn (1964).

else

Selecting the 2nd largest element takes at most f2(0,1)=2 further comparisons Kislitsyn (1964).

end

□

Case 2. *Let P be the partially ordered set as visualised in Figure 4. The 4-th largest element of P can be computed by at most 4 comparisons.*

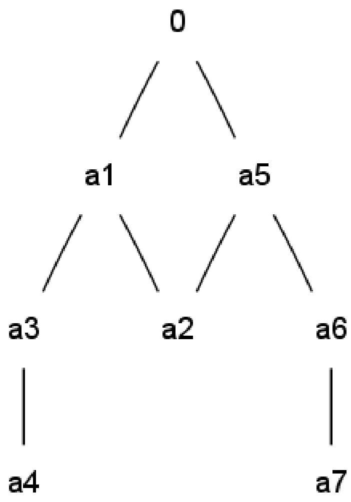

Figure 4: Poset P

Proof. Compare a3 and a6.
if *a3 > a6* **then**
Compare a2 and a4.
if *a2 > a4* **then**
Selecting the 2nd largest element takes at most f2(0,1)=2 further comparisons Kislitsyn (1964).
else
Selecting the 2nd largest element takes at most f2(0,1)=2 further comparisons Kislitsyn (1964).
end
else
The case is isomorphic to the situation a3 < a6.
end

□

References

AIGNER, M., 1982: *Selecting the top three elements*. Discrete Applied Mathematics, **4**, pp. 247–267. 2

KISLITSYN, S. S., 1964: *On the selection of the k-th element of an ordered set by pairwise comparisons*. Sib Mat Z, **5**, pp. 557–564. 2, 3